

Series JBB/5
SET-1


 कोड नं. **30/5/1**
 Code No.

 रोल नं.
 Roll No.


परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।	(I) Please check that this question paper contains 23 printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।	(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 40 प्रश्न हैं ।	(III) Please check that this question paper contains 40 questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।	(IV) Please write down the Serial Number of the question in the answer-book before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित (मानक) - सैद्धान्तिक 
MATHEMATICS (STANDARD) – Theory

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) यह प्रश्न-पत्र चार खण्डों में विभाजित किया गया है – क, ख, ग एवं घ । इस प्रश्न-पत्र में 40 प्रश्न हैं । सभी प्रश्न अनिवार्य हैं ।
- (ii) खण्ड क में प्रश्न संख्या 1 से 20 तक 20 प्रश्न हैं एवं प्रत्येक प्रश्न एक अंक का है ।
- (iii) खण्ड ख में प्रश्न संख्या 21 से 26 तक 6 प्रश्न हैं एवं प्रत्येक प्रश्न दो अंकों का है ।
- (iv) खण्ड ग में प्रश्न संख्या 27 से 34 तक 8 प्रश्न हैं एवं प्रत्येक प्रश्न तीन अंकों का है ।
- (v) खण्ड घ में प्रश्न संख्या 35 से 40 तक 6 प्रश्न हैं एवं प्रत्येक प्रश्न चार अंकों का है ।
- (vi) प्रश्न-पत्र में समग्र पर कोई विकल्प नहीं है । तथापि एक-एक अंक वाले दो प्रश्नों में, दो-दो अंकों वाले दो प्रश्नों में, तीन-तीन अंकों वाले तीन प्रश्नों में, चार-चार अंकों वाले तीन प्रश्नों में आंतरिक विकल्प दिए गए हैं । ऐसे प्रश्नों में केवल एक ही विकल्प का उत्तर लिखिए ।
- (vii) इसके अतिरिक्त, आवश्यकतानुसार, प्रत्येक खण्ड और प्रश्न के साथ यथोचित निर्देश दिए गए हैं ।
- (viii) कैलकुलेटर के प्रयोग की अनुमति नहीं है ।

खण्ड क

प्रश्न संख्या 1 से 20 तक प्रत्येक प्रश्न 1 अंक का है ।

प्रश्न संख्या 1 से 10 तक बहुविकल्पीय प्रश्न हैं ।

सही विकल्प चुनिए ।

1. बहुपद $p(x)$ को $x^2 - 4$ से विभाजित करने पर भागफल तथा शेषफल क्रमशः x तथा 3 पाए गए । बहुपद $p(x)$ है
 - (A) $3x^2 + x - 12$
 - (B) $x^3 - 4x + 3$
 - (C) $x^2 + 3x - 4$
 - (D) $x^3 - 4x - 3$

**General Instructions :**

Read the following instructions very carefully and strictly follow them :

- (i) This question paper comprises **four** sections – A, B, C and D. This question paper carries **40** questions. All questions are **compulsory**.
- (ii) **Section A** : Question Numbers **1 to 20** comprises of **20** questions of **one** mark each.
- (iii) **Section B** : Question Numbers **21 to 26** comprises of **6** questions of **two** marks each.
- (iv) **Section C** : Question Numbers **27 to 34** comprises of **8** questions of **three** marks each.
- (v) **Section D** : Question Numbers **35 to 40** comprises of **6** questions of **four** marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark, 2 questions of two marks, 3 questions of three marks and 3 questions of four marks. You have to attempt only one of the choices in such questions.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is **not** permitted.

SECTION A

Question numbers 1 to 20 carry 1 mark each.

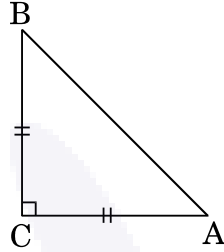
Question numbers 1 to 10 are multiple choice questions.

Choose the correct option.

1. On dividing a polynomial $p(x)$ by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial $p(x)$ is
- (A) $3x^2 + x - 12$
 - (B) $x^3 - 4x + 3$
 - (C) $x^2 + 3x - 4$
 - (D) $x^3 - 4x - 3$

2. आकृति-1 में, ABC एक समद्विबाहु समकोण त्रिभुज है जिसका समकोण C पर है। अतः

- (A) $AB^2 = 2AC^2$
- (B) $BC^2 = 2AB^2$
- (C) $AC^2 = 2AB^2$
- (D) $AB^2 = 4AC^2$



आकृति-1

3. x-अक्ष पर स्थित वह बिंदु जो $(-4, 0)$ तथा $(10, 0)$ से समदूरस्थ है, के निर्देशांक हैं

- (A) $(7, 0)$
- (B) $(5, 0)$
- (C) $(0, 0)$
- (D) $(3, 0)$

अथवा

एक वृत्त के केन्द्र के निर्देशांक, जिसके एक व्यास के अंत्य बिंदु $(-6, 3)$ और $(6, 4)$ हैं, होंगे

- (A) $(8, -1)$
- (B) $(4, 7)$
- (C) $\left(0, \frac{7}{2}\right)$
- (D) $\left(4, \frac{7}{2}\right)$

4. यदि द्विघात समीकरण $2x^2 + kx + 2 = 0$ के मूल समान हों, तो k का मान है

- (A) 4
- (B) ± 4
- (C) -4
- (D) 0



2. In Figure-1, ABC is an isosceles triangle, right-angled at C. Therefore

- (A) $AB^2 = 2AC^2$
- (B) $BC^2 = 2AB^2$
- (C) $AC^2 = 2AB^2$
- (D) $AB^2 = 4AC^2$

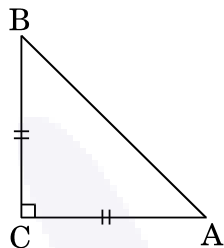


Figure-1

3. The point on the x-axis which is equidistant from $(-4, 0)$ and $(10, 0)$ is

- (A) $(7, 0)$
- (B) $(5, 0)$
- (C) $(0, 0)$
- (D) $(3, 0)$

OR

The centre of a circle whose end points of a diameter are $(-6, 3)$ and $(6, 4)$ is

- (A) $(8, -1)$
- (B) $(4, 7)$
- (C) $\left(0, \frac{7}{2}\right)$
- (D) $\left(4, \frac{7}{2}\right)$

4. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

- (A) 4
- (B) ± 4
- (C) -4
- (D) 0

5. निम्नलिखित में से कौन-सी समांतर श्रेणी *नहीं* है ?

(A) $-1 \cdot 2, 0 \cdot 8, 2 \cdot 8, \dots$

(B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

(D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \dots$

6. रैखिक समीकरणों $\frac{3x}{2} + \frac{5y}{3} = 7$ तथा $9x + 10y = 14$ का युग्म

(A) संगत है

(B) असंगत है

(C) संगत है तथा सिर्फ एक हल है

(D) संगत है तथा अनेक हल हैं

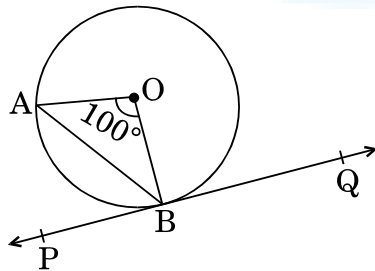
7. आकृति-2 में, O केन्द्र वाले वृत्त पर, बिंदु B पर स्पर्श-रेखा PQ खींची गई है। यदि $\angle AOB = 100^\circ$ है, तो $\angle ABP$ बराबर है

(A) 50°

(B) 40°

(C) 60°

(D) 80°



आकृति-2



5. Which of the following is **not** an A.P. ?

(A) $-1.2, 0.8, 2.8, \dots$

(B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

(D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \dots$

6. The pair of linear equations

$$\frac{3x}{2} + \frac{5y}{3} = 7 \quad \text{and} \quad 9x + 10y = 14 \quad \text{is}$$

(A) consistent

(B) inconsistent

(C) consistent with one solution

(D) consistent with many solutions

7. In Figure-2, PQ is tangent to the circle with centre at O, at the point B. If

$\angle AOB = 100^\circ$, then $\angle ABP$ is equal to

(A) 50°

(B) 40°

(C) 60°

(D) 80°

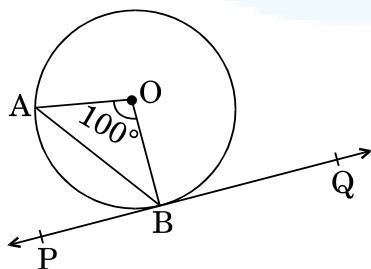
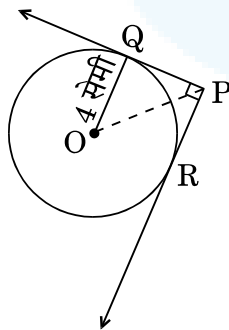


Figure-2

8. 12π घन सेमी आयतन वाले गोले की त्रिज्या (सेमी में) है
- (A) 3
(B) $3\sqrt{3}$
(C) $3^{2/3}$
(D) $3^{1/3}$
9. बिंदुओं $(m, -n)$ तथा $(-m, n)$ के बीच की दूरी है
- (A) $\sqrt{m^2 + n^2}$
(B) $m + n$
(C) $2\sqrt{m^2 + n^2}$
(D) $\sqrt{2m^2 + 2n^2}$
10. आकृति-3 में, O केन्द्र वाले वृत्त पर बाह्य बिंदु P से दो स्पर्श-रेखाएँ PQ तथा PR खींची गई हैं। वृत्त की त्रिज्या 4 सेमी है। यदि $\angle QPR = 90^\circ$ है, तो PQ की लम्बाई होगी
- (A) 3 सेमी
(B) 4 सेमी
(C) 2 सेमी
(D) $2\sqrt{2}$ सेमी



आकृति-3



8. The radius of a sphere (in cm) whose volume is $12\pi \text{ cm}^3$, is
- (A) 3
(B) $3\sqrt{3}$
(C) $3^{2/3}$
(D) $3^{1/3}$
9. The distance between the points $(m, -n)$ and $(-m, n)$ is
- (A) $\sqrt{m^2 + n^2}$
(B) $m + n$
(C) $2\sqrt{m^2 + n^2}$
(D) $\sqrt{2m^2 + 2n^2}$
10. In Figure-3, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90^\circ$, then length of PQ is
- (A) 3 cm
(B) 4 cm
(C) 2 cm
(D) $2\sqrt{2}$ cm

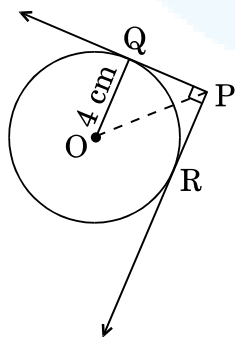


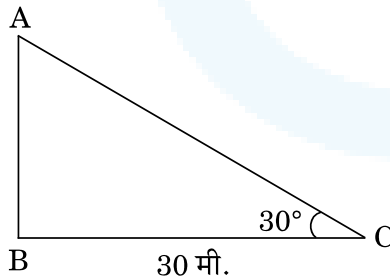
Figure-3

प्रश्न संख्या 11 से 15 में रिक्त स्थान भरिए ।

11. एक निश्चित घटने वाली घटना की प्रायिकता _____ होती है ।
12. सरलतम रूप में $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \underline{\hspace{2cm}}$ है ।
13. AOBC एक आयत है जिसके तीन शीर्ष-बिंदु $A(0, -3)$, $O(0, 0)$ एवं $B(4, 0)$ हैं । इसके विकर्ण की लम्बाई _____ है ।
14. सूत्र $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$ में, $u_i = \underline{\hspace{2cm}}$ ।
15. सभी संकेन्द्री वृत्त परस्पर _____ होते हैं ।

प्रश्न संख्या 16 से 20 में निम्नलिखित के उत्तर दीजिए ।

16. प्रथम 100 प्राकृत संख्याओं का योगफल ज्ञात कीजिए ।
17. आकृति-4 में, भूमि के एक बिंदु C से, जो मीनार के पाद-बिंदु से 30 मी. दूर है, एक मीनार के शिखर का उन्नयन कोण 30° है । मीनार की ऊँचाई ज्ञात कीजिए ।



आकृति-4

18. दो संख्याओं का ल.स. (LCM) 182 है तथा उनका म.स. (HCF) 13 है । यदि एक संख्या 26 है, तो दूसरी संख्या ज्ञात कीजिए ।

Fill in the blanks in question numbers 11 to 15.

11. The probability of an event that is sure to happen, is _____ .
12. Simplest form of $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is _____ .
13. AOBC is a rectangle whose three vertices are A(0, -3), O(0, 0) and B(4, 0). The length of its diagonal is _____ .
14. In the formula $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$, $u_i =$ _____ .
15. All concentric circles are _____ to each other.

Answer the following question numbers 16 to 20.

16. Find the sum of the first 100 natural numbers.
17. In Figure-4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30° . Find the height of the tower.

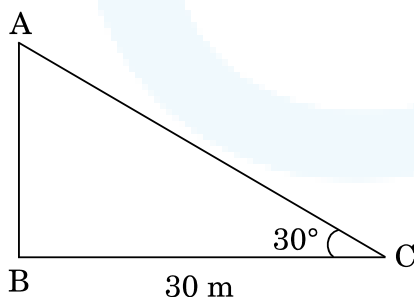


Figure-4

18. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

19. एक द्विघात बहुपद ज्ञात कीजिए जिसके शून्यकों का योगफल तथा गुणनफल क्रमशः (-3) तथा 2 हैं।

अथवा

क्या यह संभव है कि बहुपद $x^4 - 3x^2 + 5x - 9$ को $(x^2 + 3)$ से विभाजित करने पर शेषफल $(x^2 - 1)$ हो ? अपने उत्तर का कारण दीजिए।

20. मान ज्ञात कीजिए :

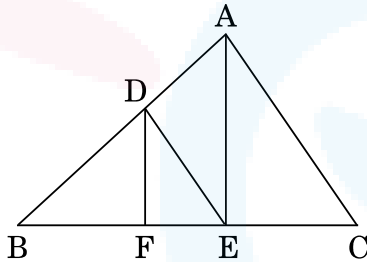
$$\frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ}$$

खण्ड ख

प्रश्न संख्या 21 से 26 तक प्रत्येक प्रश्न 2 अंकों का है।

21. दी गई आकृति-5 में, $DE \parallel AC$ है तथा $DF \parallel AE$ है। सिद्ध कीजिए कि

$$\frac{BF}{FE} = \frac{BE}{EC}$$



आकृति-5

22. दर्शाइए कि संख्या $5 + 2\sqrt{7}$ एक अपरिमेय संख्या है, जहाँ दिया गया है कि $\sqrt{7}$ एक अपरिमेय संख्या है।

अथवा

जाँच कीजिए कि क्या किसी प्राकृत संख्या n के लिए, संख्या 12^n अंक 0 पर समाप्त हो सकती है।

23. यदि A, B तथा C किसी $\triangle ABC$ के आंतरिक कोण हैं, तो सिद्ध कीजिए कि

$$\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right).$$

19. Form a quadratic polynomial, the sum and product of whose zeroes are (-3) and 2 respectively.

OR

Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$?
Justify your answer with reasons.

20. Evaluate :

$$\frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ}$$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. In the given Figure-5, $DE \parallel AC$ and $DF \parallel AE$.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

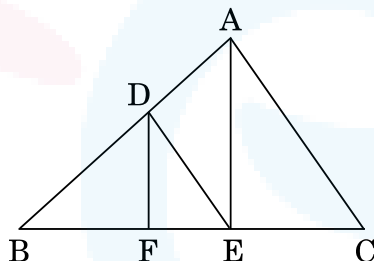


Figure-5

22. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

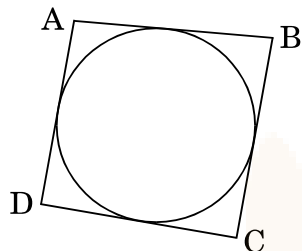
OR

Check whether 12^n can end with the digit 0 for any natural number n .

23. If A , B and C are interior angles of a ΔABC , then show that

$$\cos \left(\frac{B + C}{2} \right) = \sin \left(\frac{A}{2} \right).$$

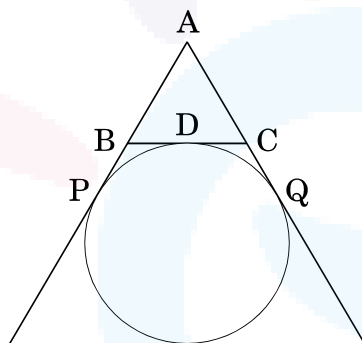
24. आकृति-6 में, एक वृत्त के परिगत एक चतुर्भुज ABCD खींचा गया है। सिद्ध कीजिए कि
 $AB + CD = BC + AD$.



आकृति-6

अथवा

- आकृति-7 में, ΔABC का परिमाण ज्ञात कीजिए, यदि $AP = 12$ सेमी है।



आकृति-7

25. निम्नलिखित बंटन का बहुलक ज्ञात कीजिए :

प्राप्तांक :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
छात्रों की संख्या :	4	6	7	12	5	6

26. दो घनों, जिनमें प्रत्येक का आयतन 125 घन सेमी है, के संलग्न फलकों को मिलाकर एक घनाभ बनाया जाता है। इस प्राप्त घनाभ का पृष्ठीय क्षेत्रफल ज्ञात कीजिए।

24. In Figure-6, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that

$$AB + CD = BC + AD.$$

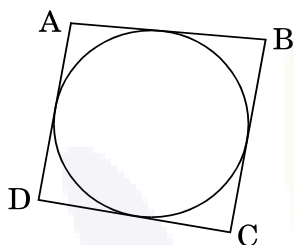


Figure-6

OR

- In Figure-7, find the perimeter of $\triangle ABC$, if $AP = 12$ cm.

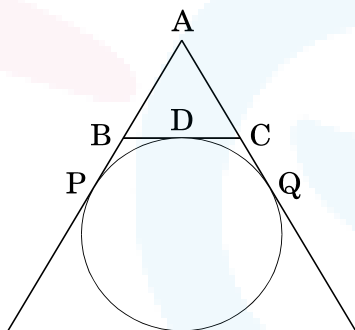


Figure-7

25. Find the mode of the following distribution :

Marks :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of Students :	4	6	7	12	5	6

26. 2 cubes, each of volume 125 cm^3 , are joined end to end. Find the surface area of the resulting cuboid.

खण्ड ग

प्रश्न संख्या 27 से 34 तक प्रत्येक प्रश्न 3 अंकों का है ।

27. यदि किसी भिन्न के अंश में से 1 घटाया जाए तो भिन्न $\frac{1}{3}$ हो जाती है तथा इसके हर में 8 जोड़ने पर भिन्न $\frac{1}{4}$ हो जाती है । भिन्न ज्ञात कीजिए ।

अथवा

एक पिता की वर्तमान आयु, अपने पुत्र की आयु के तीन गुने से तीन वर्ष अधिक है । तीन वर्ष के बाद पिता की आयु पुत्र की आयु के दुगुने से 10 वर्ष अधिक होगी । उनकी वर्तमान आयु ज्ञात कीजिए ।

28. यूक्लिड विभाजन प्रमेयिका का प्रयोग करके दर्शाइए कि किसी धनात्मक पूर्णांक का वर्ग, किसी पूर्णांक q के लिए $3q$ या $3q + 1$ के रूप का होता है ।
29. बिंदुओं $(6, -4)$ तथा $(-2, -7)$ को जोड़ने वाले रेखाखंड को y -अक्ष किस अनुपात में विभाजित करती है ? इस प्रतिच्छेद बिंदु के निर्देशांक भी ज्ञात कीजिए ।

अथवा

दर्शाइए कि बिंदु $(7, 10)$, $(-2, 5)$ तथा $(3, -4)$ एक समद्विबाहु समकोण त्रिभुज के शीर्ष-बिंदु हैं ।

30. सिद्ध कीजिए कि :

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

31. किसी समांतर श्रेणी के लिए दिया गया है कि प्रथम पद $(a) = 5$, सार्व अंतर $(d) = 3$, तथा n वाँ पद $(a_n) = 50$ है । इस समांतर श्रेणी के लिए n तथा प्रथम n पदों का योगफल (S_n) ज्ञात कीजिए ।
32. एक ΔABC की रचना कीजिए जिसकी भुजाएँ $BC = 6$ सेमी, $AB = 5$ सेमी तथा $\angle ABC = 60^\circ$ हैं । फिर एक ऐसे त्रिभुज की रचना कीजिए जिसकी भुजाएँ ΔABC की संगत भुजाओं की $\frac{3}{4}$ गुनी हों ।

अथवा

3.5 सेमी त्रिज्या का एक वृत्त खींचिए । वृत्त के केंद्र से 7 सेमी की दूरी पर किसी बाह्य बिंदु P से इस वृत्त पर दो स्पर्श-रेखाओं की रचना कीजिए ।

**SECTION C**

Question numbers 27 to 34 carry 3 marks each.

27. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

28. Use Euclid Division Lemma to show that the square of any positive integer is either of the form $3q$ or $3q + 1$ for some integer q .
29. Find the ratio in which the y-axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

OR

Show that the points $(7, 10)$, $(-2, 5)$ and $(3, -4)$ are vertices of an isosceles right triangle.

30. Prove that :

$$\frac{\sqrt{1 + \sin A}}{\sqrt{1 - \sin A}} = \sec A + \tan A$$

31. For an A.P., it is given that the first term $(a) = 5$, common difference $(d) = 3$, and the n^{th} term $(a_n) = 50$. Find n and sum of first n terms (S_n) of the A.P.
32. Construct a ΔABC with sides $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60^\circ$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC .

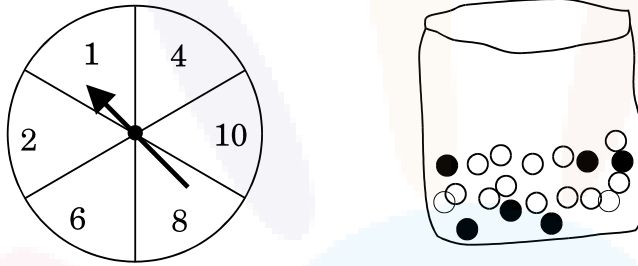
OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

33. निम्नलिखित अनुच्छेद को पढ़कर अंत में दिए गए प्रश्नों के उत्तर दीजिए :

दिवाली मेला

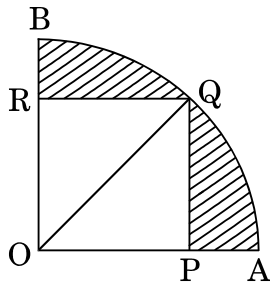
दिवाली मेले के एक बूथ के एक खेल में पहले एक स्पिनर का प्रयोग किया जाता है और उसके बाद यदि स्पिनर एक सम संख्या पर रुकता है, तो खिलाड़ी को एक थैले में से एक कंचा चुनने दिया जाता है। स्पिनर तथा थैले में कंचे दी गई आकृति-8 में दिखाए गए हैं। यदि काले रंग का कंचा चुना जाता है, तो इनाम दिए जाते हैं। श्वेता एक बार खेल खेलती है।



आकृति-8

- (i) प्रायिकता क्या है कि उसे थैले में से कंचा चुनने दिया जाएगा ?
- (ii) माना उसे थैले में से कंचा चुनने दिया जाता है, तो उसके इनाम पाने की प्रायिकता क्या है, जब दिया गया है कि थैले में 20 कंचे हैं जिनमें से 6 काले हैं ?

34. आकृति-9 में, एक वृत्त का चतुर्थांश OAQB के अंतर्गत एक वर्ग OPQR बना हुआ है। यदि वृत्त की त्रिज्या $6\sqrt{2}$ सेमी हो, तो छायांकित भाग का क्षेत्रफल ज्ञात कीजिए।



कृति-9

33. Read the following passage and answer the questions given at the end :

Diwali Fair

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure-8.

Prizes are given, when a black marble is picked. Shweta plays the game once.

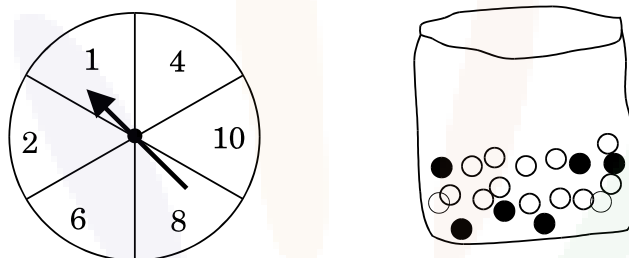


Figure-8

- (i) What is the probability that she will be allowed to pick a marble from the bag ?
- (ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black ?
34. In Figure-9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region.

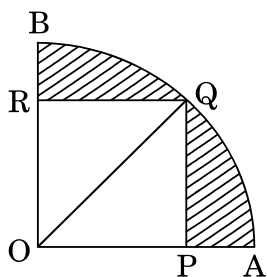


Figure-9

खण्ड घ

प्रश्न संख्या 35 से 40 तक प्रत्येक प्रश्न 4 अंकों का है ।

35. बहुपद $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ के दो शून्यक $\sqrt{5}$ तथा $-\sqrt{5}$ हैं । इस बहुपद के अन्य दो शून्यक ज्ञात कीजिए ।

अथवा

बहुपद $2x^3 - 3x^2 + 6x + 7$ में कम-से-कम क्या जोड़ा जाए कि प्राप्त बहुपद $x^2 - 4x + 8$ से पूर्णतया विभाजित हो जाए ?

36. सिद्ध कीजिए कि दो समरूप त्रिभुजों के क्षेत्रफलों का अनुपात इनकी संगत भुजाओं के अनुपात के वर्ग के बराबर होता है ।
37. दो वर्गों के क्षेत्रफलों का योगफल 544 वर्ग मी. है । यदि उनके परिमापों में 32 मी. का अन्तर हो, तो दोनों वर्गों की भुजाएँ ज्ञात कीजिए ।

अथवा

एक मोटरबोट, जिसकी स्थिर जल में चाल 18 किमी/घंटा है, 24 किमी धारा के प्रतिकूल जाने में, वही दूरी धारा के अनुकूल जाने की अपेक्षा 1 घंटा अधिक लेती है । धारा की चाल ज्ञात कीजिए ।

38. एक ठोस खिलौना 7 सेमी आधार त्रिज्या वाले एक लम्ब-वृत्तीय शंकु के आकार का है जो उसी त्रिज्या वाले एक अर्धगोले पर अध्यारोपित है । यदि शंकु की ऊँचाई 10 सेमी हो, तो खिलौने का आयतन ज्ञात कीजिए । उस रंगीन कागज़ का क्षेत्रफल भी ज्ञात कीजिए जिससे इस खिलौने को पूरी तरह से ढँका जा सके । ($\pi = \frac{22}{7}$ तथा $\sqrt{149} = 12.2$ प्रयोग कीजिए)
39. एक पेडस्टल के शिखर पर 1.6 मी. ऊँची मूर्ति लगी हुई है । भूमि के एक बिंदु से मूर्ति के शिखर का उन्नयन कोण 60° है और उसी बिंदु से पेडस्टल के शिखर का उन्नयन कोण 45° है । पेडस्टल की ऊँचाई ज्ञात कीजिए । ($\sqrt{3} = 1.73$ प्रयोग कीजिए)

**SECTION D**

Question numbers 35 to 40 carry 4 marks each.

35. Obtain other zeroes of the polynomial

$$p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$$

if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?

36. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
37. Sum of the areas of two squares is 544 m^2 . If the difference of their perimeters is 32 m, find the sides of the two squares.

OR

A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

38. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy. (Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)
39. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal. (Use $\sqrt{3} = 1.73$)

40. निम्नलिखित आँकड़ों के लिए 'से कम' प्रकार का तोरण खींचिए। अतः बंटन का माध्यक ज्ञात कीजिए।

आयु (वर्षों में) :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
व्यक्तियों की संख्या :	5	15	20	25	15	11	9

अथवा

नीचे दिया गया बंटन एक-दिवसीय क्रिकेट मैचों में, गेंदबाजों द्वारा लिए गए विकेटों की संख्या दर्शाता है। लिए गए विकेटों की संख्या का माध्य तथा माध्यक ज्ञात कीजिए।

विकेटों की संख्या :	20 – 60	60 – 100	100 – 140	140 – 180	180 – 220	220 – 260
गेंदबाजों की संख्या :	7	5	16	12	2	3



40. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (in years) :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of persons :	5	15	20	25	15	11	9

OR

The distribution given below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20 – 60	60 – 100	100 – 140	140 – 180	180 – 220	220 – 260
Number of bowlers :	7	5	16	12	2	3

CBSE Board Paper Solution-2020

Class	: X
Subject	: Mathematics (Standard) - Theory
Set	: 1
Code No	: 30/5/1
Time allowed	: 3 Hours
Maximum Marks	: 80 Marks

General instructions:

Read the following instructions very carefully and strictly follow them:

- (i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 question All questions are compulsory
- (ii) **Section A:** Question Numbers 1 to 20 comprises of 20 question of one mark each.
- (iii) **Section B:** Question Numbers 21 to 26 comprises of 6 question of two marks each.
- (iv) **Section C:** Question Numbers 27 to 34 comprises of 8 question of three marks each.
- (v) **Section D:** Question Numbers 35 to 40 comprises of 6 question of four marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 question of the mark, 2 question of one mark, 2 questions of two marks. 3 question of three marks



and 3 question of four marks. You have to attempt only one of the choices in such questions.

(vii) In addition to this. Separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculations is not permitted.

Section A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice questions.

Choose the correct option.

1. On dividing a polynomial $p(x)$ by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial $p(x)$ is

(A) $3x^2 + x - 12$

(B) $x^3 - 4x + 3$

(C) $x^2 + 3x - 4$

(D) $x^2 - 4x - 3$

Answer:

Correct Answer: (B) $x^3 - 4x + 3$

Explanation:

$$\begin{aligned} P(x) &= (\text{divisor}) \times (\text{quotient}) + \text{Remainder} \\ &= (x^2 - 4)x + 3 \\ &= x^3 - 4x + 3 \end{aligned}$$

2) In Figure-1, ABC is an isosceles triangle, right-angled at C. Therefore



- (A) $AB^2 = 2AC^2$
- (B) $BC^2 = 2AB^2$
- (C) $AC^2 = 2AB^2$
- (D) $AB^2 = 4AC^2$

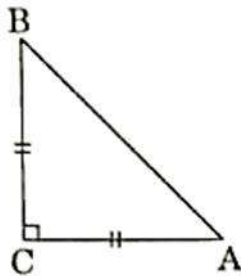


Figure-1

Answer:

Correct Answer: (A) $AB^2 = 2AC^2$

Explanation:

Given that $\triangle ACB$ is an isosceles triangle right angled at C .

Therefore, $AC = BC$

Using Pythagoras theorem in the given triangle, we have

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= AC^2 + AC^2 \\ &= 2AC^2 \end{aligned}$$

- 3) The point on the x-axis which is equidistant from $(-4, 0)$ and $(10, 0)$ is**
- (A) $(7, 0)$**



(B) (5, 0)

(C) (0, 0)

(D) (3, 0)

OR

The centre of a circle whose end points of a diameter are $(-6, 3)$ and $(6, 4)$ is

(A) (8, -1)

(B) (4, 7)

(C) $\left(0, \frac{7}{2}\right)$

(D) $\left(4, \frac{7}{2}\right)$

Answer:

Correct Answer: (D) (3, 0)

Explanation:

The required point and the given points as well lie on the x-axis.

The required point $(x, 0)$ is the mid-point of the line joining points $(-4, 0)$ and $(10, 0)$.

$$\begin{aligned}\text{So,} \quad x &= (-4+10)/2 \\ &= 6/2 \\ &= 3\end{aligned}$$

$$\begin{aligned}\text{Required point} &= (x, 0) \\ &= (3, 0)\end{aligned}$$



OR

Correct Answer: (C) (0, 7/2)

Explanation:

The centre of a circle is the mid-point of its diameter.

End points of the diameter are: $(-6, 3)$ and $(6, 4)$

Coordinates of the centre = $\left(\frac{-6+6}{2}, \frac{3+4}{2}\right)$
 $= (0, 7/2)$

4) The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(A) 4

(B) ± 4

(C) -4

(D) 0

Answer:

Correct Answer: (B) ± 4

Explanation:

The given equation is:

$$2x^2 + kx + 2 = 0$$

$$\text{Discriminant} = b^2 - 4ac$$

$$\text{Here, } b = k, a = 2, \text{ and } c = 2$$

$$\begin{aligned} \text{So, Discriminant} &= k^2 - 4 \times 2 \times 2 \\ &= k^2 - 16 \end{aligned}$$

A quadratic equation has equal roots if its discriminant is zero.

$$k^2 - 16 = 0$$

$$k^2 = 16$$

$$k = \pm 4$$

5) Which of the following is not an A.P.?

(A) $-1.2, 0.8, 2.8, \dots$

(B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

(D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \dots$

Answer:

Correct Answer: (C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

Explanation:



$$\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$$

$$\frac{7}{3} - \frac{4}{3} = \frac{7-4}{3}$$

$$= \frac{3}{3}$$

$$= 1$$

$$\frac{9}{3} - \frac{7}{3} = \frac{9-7}{3}$$

$$= \frac{2}{3}$$

$$\Rightarrow \frac{3}{3} \neq \frac{2}{3}$$

Difference between consecutive terms is not same. So, this is not an A.P.

6) The pair of linear equations

$$\frac{3x}{2} + \frac{5y}{3} = 7 \text{ and } 9x + 10y = 14 \text{ is}$$

(A) consistent

(B) inconsistent

(C) consistent with one solution

(D) consistent with many solutions

Answer:

Correct Answer: (B) Inconsistent

Explanation:



$$\frac{3x}{2} + \frac{5y}{3} = 7$$

$$\frac{9x + 10y}{6} = 7$$

$$9x + 10y = 42 \quad \dots(1)$$

$$9x + 10y = 14 \quad \dots(2)$$

Ratios of coefficients of x and that of y are

$$\frac{9}{9} = \frac{10}{10} = \frac{1}{1}$$

$$\text{Ratio of constants} = \frac{42}{14} = \frac{3}{1} \neq \frac{1}{1}$$

Ratios of coefficients of x and y are equal

but they are not equal to the ratio of constants.

So, the given equations represent a pair of parallel lines and so they do not have a common solution.

7) In Figure-2 PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100^\circ$, then $\angle ABP$ is equal to

(A) 50°

(B) 40°

(C) 60°

(D) 80°

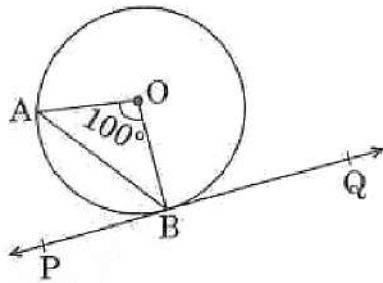


Figure-2

Answer:

Correct Answer: (A) 50°

Explanation:

$OA = OB$ (radii)

So, $\angle OAB = \angle OBA$

$$= (180^\circ - 100^\circ)/2$$

$$= 40^\circ$$

Now, a radius of a circle meets a tangent at 90° .

So, $\angle ABP = \angle OBP - \angle OBA$

$$= 90^\circ - 40^\circ = 50^\circ$$

8) The radius of a sphere (in cm) whose volume is $12\pi \text{ cm}^3$, is

(A) 3

(B) $3\sqrt{3}$

(C) $3^{2/3}$

(D) $3^{1/3}$



Answer:

Correct Answer: (C) $3^{2/3}$

Explanation:

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$12\pi = \frac{4}{3} \pi r^3$$

$$r^3 = 3^2$$

$$r = 3^{2/3}$$

9) The distance between the points $(m, -n)$ and $(-m, n)$ is

(A) $\sqrt{m^2 + n^2}$

(B) $m + n$

(C) $2\sqrt{m^2 + n^2}$

(D) $\sqrt{2m^2 + 2n^2}$

Answer:

Correct Answer: (C) $2\sqrt{m^2 + n^2}$

Explanation:

$$\text{Distance} = \sqrt{m - (-m)^2 + (-n - n)^2}$$

$$= \sqrt{(m + m)^2 + (-2n)^2}$$

$$= 2\sqrt{m^2 + n^2}$$



10) In Figure-3. From an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90^\circ$, then length of PQ is

- (A) 3cm
- (B) 4cm
- (C) 2cm
- (D) $2\sqrt{2}$ cm

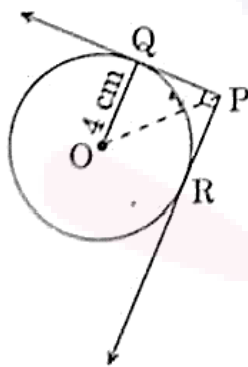


Figure-3

Answer:

Correct Answer: (B) 4 cm

Explanation:

Tangents are drawn from an external point P.

So, line joining centre O and point P bisects $\angle QPR$.

OP bisects $\angle QPR = 90^\circ$.

In ΔOQP ,

$\angle Q = 90^\circ$ (radius meets tangent at 90°)

$\angle QPO = 45^\circ = \angle QOP$



Thus, $OQ = PQ = 4$ cm

Fill in the blanks in question number 11 to 15

11) The probability of an event that is sure to happen is ____.

Answer: 1

12) Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is ____.

Answer:

$\cot^2 A$

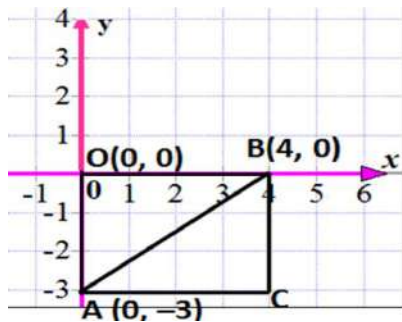
$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A} = \cot^2 A$$

13) AOBC is a rectangle whose three vertices are $A(0, -3)$, $O(0, 0)$ and $B(4, 0)$. The length of its diagonal is ____.

Answer:

In right-angled triangle AOB,

$$AB = \sqrt{OA^2 + OB^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



14) In the formula $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$, $u_i = \underline{\hspace{2cm}}$.

Answer:

$$\frac{x_i - a}{h}$$

15) All concentric circles are to each other.

Answer: similar

Answer the following question numbers 16 to 20.

16) Find the sum of the first 100 natural numbers.

Answer:

$1 + 2 + 3 + \dots + 100$ is an A. P.

Here first term $a = 1$

Common difference $d = 1$

Sum of n terms of an A.P. $= \frac{n}{2} [2a + (n-1)d]$

The sum of first 100 natural numbers

$$= \frac{100}{2} [2 \times 1 + (100 - 1) \times 1]$$

$$= \frac{100 (101)}{2}$$

$$= 50 \times 101$$

$$= 5050$$

17) In Figure-4 the angle of elevation of the top of a tower from a point C on the ground, which is



**30 m away from the foot of the tower, is 30° .
Find the height of the tower.**

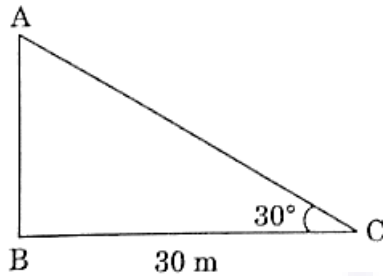


Figure-4

Answer:

$$\tan 30^\circ = \frac{AB}{30}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

So, the height of the tower is $10\sqrt{3}$ m.

18) The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, Find the other.

Answer:

LCM \times HCF = Product of the two numbers

$$182 \times 13 = 26 \times x$$

$$x = \frac{182 \times 13}{26} = 91$$

So, the other number is 91.



19) Form a quadratic polynomial, the sum and product of whose zeroes are (-3) and 2 respectively.

OR

Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$? Justify your answer with reasons.

Answer:

$$\begin{aligned} & x^2 - (\text{sum of zeroes})x + \text{product of zeroes} \\ & = x^2 - (-3)x + 2 \\ & = x^2 + 3x + 2 \end{aligned}$$

So, the required polynomial is $x^2 + 3x + 2$.

OR

When a polynomial $p(x)$ is divided by another polynomial $g(x)$, then the degree of remainder $r(x) < \text{degree of } g(x)$

Therefore, for the given question $x^2 - 1$ cannot be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $x^2 + 3$ because $\text{deg}(x^2 - 1) = \text{deg}(x^2 + 3)$.

20) Evaluate:

$$\frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ}$$

Answer:

$$\frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ}$$
$$= \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}}$$
$$= 2$$

SECTION B

Question number 21 to 26 carry 2 marks each.

21) In the given Figure-5, $DE \parallel AC$ and $DF \parallel AE$.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

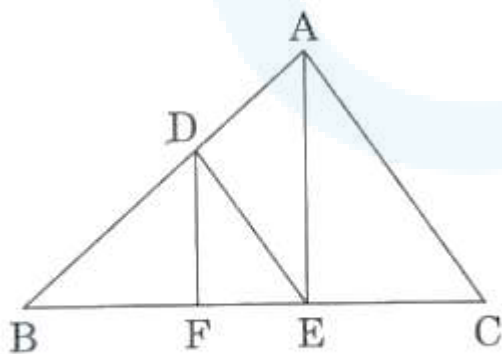


Figure-5

Answer:



In $\triangle ABC$, $DE \parallel AC$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA} = \frac{BE}{EC} \quad \dots(1)$$

In $\triangle BAE$, $DF \parallel AE$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA} = \frac{BF}{FE} \quad \dots(2)$$

From (1) and (2), we get

$$\frac{BE}{EC} = \frac{BF}{FE}$$

22) Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Answer:

Let us assume, to the contrary, that $5 + 2\sqrt{7}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that

$$5 + 2\sqrt{7} = \frac{a}{b}$$

$$\therefore 2\sqrt{7} = \frac{a}{b} - 5$$

Rearranging this equation, we get $\sqrt{7} = \frac{1}{2} \left(\frac{a}{b} - 5 \right) = \frac{a - 5b}{2b}$

Since, a and b are integers, we get $\frac{a - 5b}{2b}$ is rational, and so $\sqrt{7}$ is a rational.

But this contradicts the fact that $\sqrt{7}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 + 2\sqrt{7}$ is rational.

So, we conclude that $5 + 2\sqrt{7}$ is irrational.

OR

If the number 12^n , for any n , were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of 12^n would contain the prime 5. This is not possible

$$\therefore 12^n = (2 \times 2 \times 3)^n$$

So, the prime numbers in the factorisation of 12^n are 2 and 3.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 12^n .

So, there is no natural number n for which 12^n ends with the digit zero.

23) If A, B and C are interior angles of a $\angle ABC$, then show that

$$\cos \left(\frac{B + C}{2} \right) = \sin \frac{A}{2}.$$

Answer:

Given that A, B and C are interior angles of a triangle ABC.

$$\therefore A + B + C = 180^\circ$$

$$\text{or } A = 180^\circ - B - C$$

Now,

$$\begin{aligned}\cos\left(\frac{B+C}{2}\right) &= \sin\left(90^\circ - \frac{B+C}{2}\right) \\ &= \sin\left(\frac{180^\circ - B - C}{2}\right) \\ &= \sin\left(\frac{A}{2}\right)\end{aligned}$$

24) In Figure 6, a quadrilateral ABCD is drawn to circumscribe a circle.

Prove that

$$\mathbf{AB + CD = BC + AD.}$$

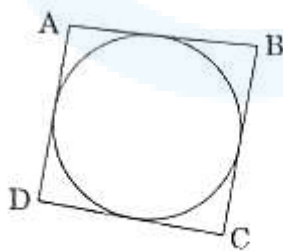


Figure-6

OR

In Figure-7, find the perimeter of $\angle ABC$, if AP = 12 cm.

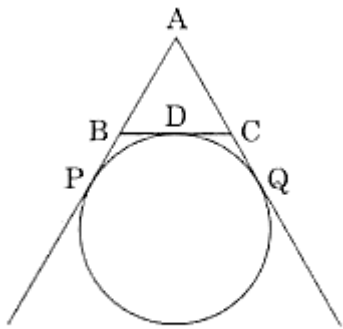
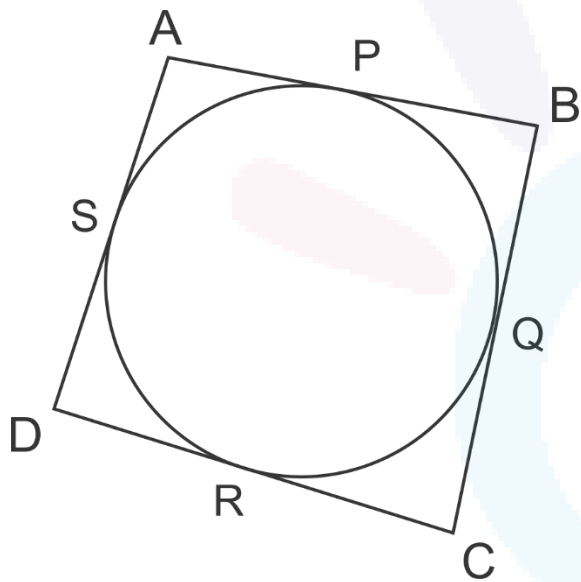


Figure-7

Answer:





We have to prove that

$$AB + CD = BC + AD$$

We know that lengths of tangents drawn from a point to a circle are equal.

Therefore, from figure, we have

$$DR = DS, CR = CQ, AS = AP, BP = BQ$$

Now,

$$\begin{aligned} \text{LHS} = AB + CD &= (AP + BP) + (CR + DR) \\ &= (AS + BQ) + (CQ + DS) \\ &= BQ + CQ + AS + DS \\ &= BC + AD \\ &= \text{RHS} \end{aligned}$$

OR

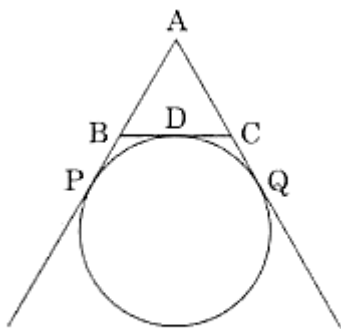


Figure-7

From the given figure, we have $AP = 12$ cm

Since AQ and AB are the tangents to the circle from a common point A , hence $AP = AQ = 12$

Similarly, $PB = BD$ and $CD = CQ$

Also, $AP = AB + PB$ and $AQ = AC + CQ$



$$\begin{aligned}\text{Perimeter of } ABC &= AB + BD + CD + AC \\ &= AB + PB + CQ + AC \\ &\quad (\text{since } PB = BM \text{ and } CM = CQ) \\ &= (AB + PB) + (CQ + AC) \\ &= AP + AQ \\ &= 12 + 12 \\ &= 24 \text{ cm}\end{aligned}$$

Therefore, the perimeter of triangle ABC = 24 cm

25) Find the mode of the following distribution:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of Students	4	6	7	12	5	6

Answer:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students	4	6	7	12	5	6



From the given data, we have

$$l = 30, f_1 = 12, f_0 = 7, f_2 = 5, h = 10$$

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 30 + \left(\frac{12 - 7}{2 \times 12 - 7 - 5} \right) \times 10 \\ &= 34.1667\end{aligned}$$

∴ Mode of the given data is 34.1667.

26) 2 cubes, each of volume 125 cm^3 , are joined end to end. Find the surface area of the resulting cuboid.

Answer:

Let the side of the old cube = a

The volume of the old cube = 125 cm^3 (Given)

The volume of the cube = a^3

$$a^3 = 125 \text{ cm}^3$$

$$a^3 = 5^3$$

$$a = 5 \text{ cm}$$

The dimensions of the resulting cuboid are:

Length, $l = 10 \text{ cm}$

Breadth, $b = 5 \text{ cm}$

Height, $h = 5 \text{ cm}$

Total surface area of the resulting cuboid:

$$= 2(lb + bh + hl)$$

$$= 2[10(5) + 5(5) + 5(10)]$$

$$= 2[50 + 25 + 50]$$

$$= 2[125]$$

$$= 250 \text{ cm}^2$$

Section C

27) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

OR

The present age of a father is three years more than three times the age of his son. Three years



hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Answer:

Let the numerator of the fraction be x and denominator be y .

Therefore, the fraction is $\frac{x}{y}$.

According to question,

$$\frac{x-1}{y} = \frac{1}{3}$$

$$3x - 1 = y$$

$$3x - 3 = y \quad \dots(1)$$

$$\text{and } \frac{x}{y+8} = \frac{1}{4}$$

$$4x = y + 8$$

$$4x - 8 = y \quad \dots(2)$$

From equations 1 and 2, we get

$$3x - 3 = 4x - 8$$

$$4x - 3x = 8 - 3$$

$$x = 5$$

Putting $x = 5$ in equation (1),

$$3 \times 5 - 3 = y$$

$$y = 12$$

So, the required fraction = $\frac{5}{12}$.

OR



Let the son's present age be x .

So, father's present age = $3x + 3$

3 years later:

Son's age = $x + 3$

Father's age = $3x + 3 + 3 = 3x + 6$

But, according to the given condition,

$$\begin{aligned} \text{3 years later father's age} &= 2x + 3 + 10 \\ &= 2x + 6 + 10 \\ &= 2x + 16 \end{aligned}$$

So, we can write

$$3x + 6 = 2x + 16$$

$$3x - 2x = 16 - 6$$

$$x = 10$$

So, son's present age = 10 years

$$\begin{aligned} \text{and father's present age} &= 10 \times 3 + 3 \\ &= 33 \text{ years} \end{aligned}$$

28) Use Euclid Division Lemma to show that the square of any positive integer is either of the form $3q$ or $3q + 1$ for some integer q .

Answer:



Let a be a positive integer and $a \div 3 = 3m + r$.

By Euclid's Algorithm,

$a = 3m + r$ for some integer $m \geq 0$ and $0 \leq r < 3$.

The possible remainders are 0, 1 and 2. Therefore, a can be $3m$ or $3m + 1$ or $3m + 2$.

Thus,

$$\begin{aligned} a^2 &= 9m^2 \text{ or } (3m+1)^2 \text{ or } (3m+2)^2 \\ &= 9m^2 \text{ or } (9m^2 + 6m + 1) \text{ or } (9m^2 + 12m + 4) \\ &= 3 \times (3m^2) \text{ or } 3(3m^2 + 2m) + 1 \text{ or } 3(3m^2 + 4m + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

where k_1, k_2 and k_3 are some positive integers.

Hence, square of any positive integer is either of the form $3q$ or $3q + 1$ for some integer q .

29) Find the ratio in which y-axis divides the line segment joining the points (6, -4) and (-2, -7). Also find the point of intersection.

OR

Show that the points (7, 10), (-2, 5) and (3, -4) are vertices of an isosceles right triangle.

Answer:



Let the ratio in which the line segment joining A(6, -4) and B(-2, -7) is divided by the y-axis be $k : 1$.

Let the coordinate of point on y-axis be $(0, y)$.

Therefore,

$$0 = \frac{-2k + 6}{k + 1} \quad \text{and} \quad y = \frac{-7k - 4}{k + 1}$$

Now,

$$0 = \frac{-2k + 6}{k + 1}$$

$$\text{or } 0 = -2k + 6$$

$$\text{or } k = 3$$

Therefore, the required ratio is 3:1.

Also,

$$\begin{aligned} y &= \frac{-7k - 4}{k + 1} \\ &= \frac{-7 \times 3 - 4}{3 + 1} \\ &= \frac{-25}{4} \end{aligned}$$

Therefore, the given line segment is divided by the point

$$\left(0, \frac{-25}{4}\right) \text{ in the ratio } 3:1.$$

OR



Let the given points are P(7, 10), Q(-2, 5) and R(3, -4).
Now, using distance formula we find distance
between these points i.e., PQ, QR and PR.

Distance between points P(7, 10) and Q(-2, 5),

$$\begin{aligned}PQ &= \sqrt{(-2 - 7)^2 + (5 - 10)^2} \\ &= \sqrt{81 + 25} \\ &= \sqrt{106}\end{aligned}$$

Distance between points Q(-2, 5) and R(3, -4),

$$\begin{aligned}QR &= \sqrt{(3 + 2)^2 + (-4 - 5)^2} \\ &= \sqrt{25 + 81} \\ &= \sqrt{106}\end{aligned}$$

Distance between points P(7, 10) and R(3, -4),

$$\begin{aligned}PR &= \sqrt{(3 - 7)^2 + (-4 - 10)^2} \\ &= \sqrt{16 + 196} \\ &= \sqrt{212}\end{aligned}$$

Now,

$$\begin{aligned}PQ^2 + QR^2 &= 106 + 106 \\ &= 212 = PR^2\end{aligned}$$

i.e., $PQ^2 + QR^2 = PR^2$

Therefore, points P(5, -2), Q(6, 4) and R(7, -2) form
an isosceles right triangle because sides PQ and QR
are equal.

30) Prove that:



$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Answer:

$$\begin{aligned} \text{LHS} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\ &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \times \sqrt{\frac{1 + \sin A}{1 + \sin A}} \\ &= (1 + \sin A) \sqrt{\frac{1}{1 - \sin^2 A}} \\ &= \frac{1 + \sin A}{\sqrt{\cos^2 A}} \\ &= \frac{1 + \sin A}{\cos A} \\ &= \frac{\sin A}{\cos A} + \frac{1}{\cos A} \\ &= \tan A + \sec A = \text{RHS} \end{aligned}$$

31) For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the nth term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P.

Answer:

Here, $a = 5$, $d = 3$, $a_n = 50$

We need to find S_n .

Firstly, we will find the value of n .

We know that

$$a_n = a + (n - 1)d$$

$$\text{So, } 50 = 5 + (n - 1)3$$

$$\text{or } 50 - 5 = (n - 1)3$$

$$\text{or } \frac{45}{3} + 1 = n$$

$$\text{or } n = 16$$

We know that sum of first n terms of an AP is given by

$$S_n = \frac{n}{2} (a + a_n)$$

$$\text{So, } S_{16} = \frac{16}{2} (5 + 50)$$

$$= 8 \times 55$$

$$\text{or } S_{16} = 440$$

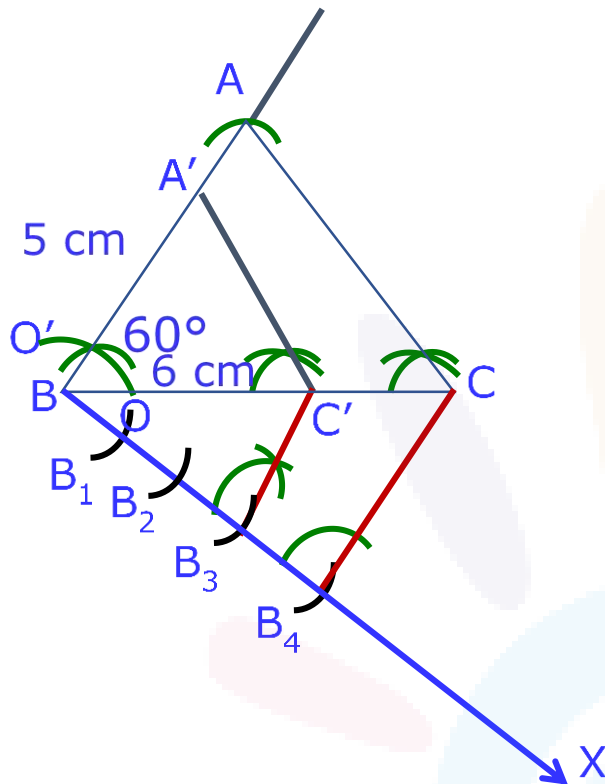
32) Construct a ΔABC with sides $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60^\circ$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC .

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents

to the circle from that point.

Answer:



Steps of Construction :

Step 1: Draw a $\triangle ABC$ with sides $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 60^\circ$.

Step 2: Draw a ray BX making an acute angle with line BC on the opposite side of vertex A .

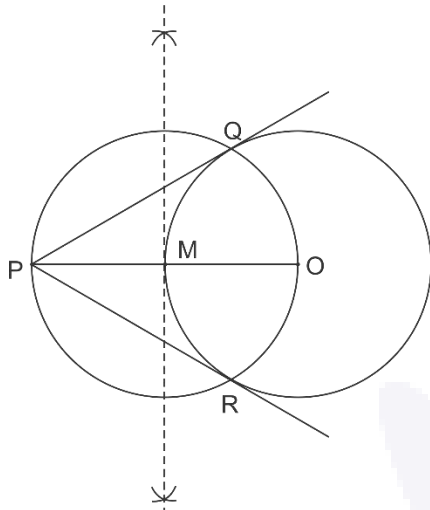
Step 3: Locate 4 points B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step 4: Join the points C and B_4 .

Step 5: Through the point B_3 , draw a line parallel to CB_4 intersecting line segment BC at point C' .

Step 6: Draw a line through C' parallel to the line AC to intersect line segment AB at A' .

The required triangle is $\triangle A'BC'$.

OR**Steps of Construction :**

Step 1: Draw a circle of radius 3.5 cm with centre at point O. Locate a point P, at a distance of 7 cm from O, and join O and P.

Step 2: Bisect OP. Let M be the mid-point of OP.

Step 3: Draw a circle with centre at M and MO as radius. Q and R are points of intersections of this circle with the circle having centre at O.

Step 4: Join PQ and PR.

PQ and PR are the required tangents.

33) Read the following passage and answer the question given at the end:

Diwali Fair.

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an

even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure – 8.

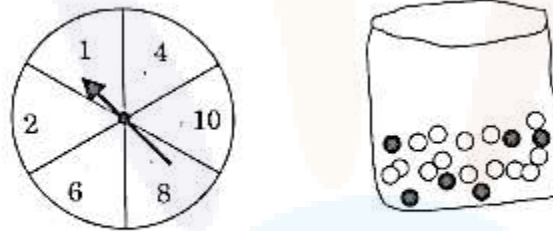


Figure-8

- (i) **What is the probability that she will be allowed to pick a marble from the bag?**
- (ii) **Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?**

Answer:



Numbers on spinner = 1, 2, 4, 6, 8, 10

Even numbers on spinner = 2, 4, 6, 8, 10

Shweta will pick black marble, if spinner stops on even number.

Therefore,

$$n(\text{Even number}) = 5$$

$$n(\text{Possible number}) = 6$$

(i) $P(\text{Shweta allowed to pick a marble})$

$$= P(\text{Even number})$$

$$= \frac{n(\text{Even number})}{n(\text{Possible number})}$$

$$= \frac{5}{6}$$

Therefore, the probability of allowing Shweta

to pick a marble is $\frac{5}{6}$.



(ii) Since, prizes are given, when a black marble is picked.

Number of black marbles = 6

Total number of marbles = 20

Therefore, $P(\text{getting a prize}) = P(\text{a black marble})$

$$\begin{aligned} &= \frac{n(\text{Black marbles})}{n(\text{Total marbles})} \\ &= \frac{6}{20} \\ &= \frac{3}{10} \end{aligned}$$

Therefore, the probability of getting prize is $\frac{3}{10}$.

34. In figure – 9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region.

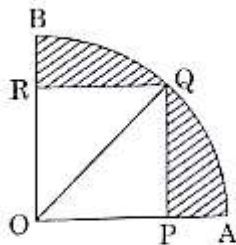


Figure-9

Answer:



Given that, $OQ = 6\sqrt{2}$ cm

OPQR is a square.

Let the side of square = a

The diagonal of square = $a\sqrt{2}$

Here, OQ is a diagonal of square.

$$\Rightarrow a\sqrt{2} = 6\sqrt{2}$$

$$\Rightarrow a = 6 \text{ cm}$$

$$\begin{aligned} \text{Area of square OPQR} &= 6^2 \\ &= 36 \text{ cm}^2 \end{aligned}$$

Radius of the quadrant OAQB = Diagonal of the square OPQR

$$= 6\sqrt{2} \text{ cm}$$

$$\begin{aligned} \text{Area of the quadrant OAQB} &= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (6\sqrt{2})^2 \\ &= \frac{396}{7} \text{ cm}^2 \end{aligned}$$

Area of shaded region = Area of the quadrant OAQB

– Area of square OPQR

$$= \frac{396}{7} - 36$$

$$= \frac{144}{7}$$

$$= 20.6 \text{ cm}^2$$

SECTION D

Obtain other zeroes of the polynomial

35) $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$

if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

OR



What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that resulting polynomial will be divisible by $x^2 - 4x + 8$?

Answer:

The given polynomial is $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$.

The two zeroes of $p(x)$ are $\sqrt{5}$ and $-\sqrt{5}$.

Therefore, $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are factors of $p(x)$.

Also, $(x - \sqrt{5})(x + \sqrt{5}) = x^2 - 5$

and so $x^2 - 5$ is a factor of $p(x)$.

Now,

$$\begin{array}{r}
 \overline{2x^2 - x - 1} \\
 x^2 - 5 \overline{) 2x^4 - x^3 - 11x^2 + 5x + 5} \\
 \underline{2x^4 } \\
 -10x^2 \\
 + \\
 \hline
 -x^3 - x^2 + 5x + 5 \\
 \underline{-x^3 + 5x} \\
 + - \\
 \hline
 -x^2 + 5 \\
 \underline{-x^2 + 5} \\
 + - \\
 \hline
 0
 \end{array}$$



()

exactly divisible by $x^2 - 4x + 8$.

Thus, we should add $-10x + 33$ to $2x^3 - 3x^2 + 6x + 7$.

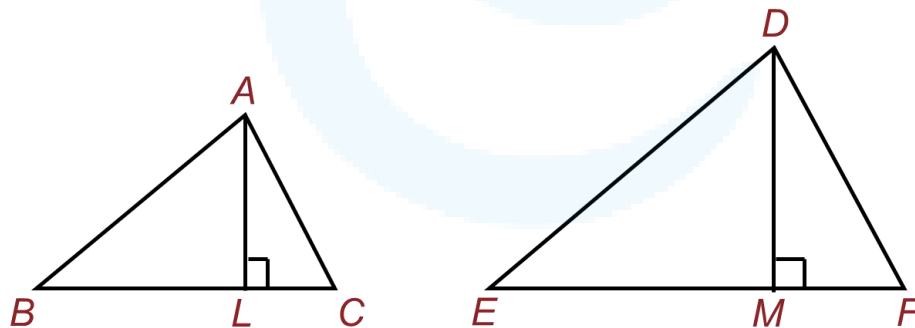
36) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Answer:

Given : $\triangle ABC \sim \triangle DEF$

To prove : $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$

Construction: Draw $AL \perp BC$ and $DM \perp EF$





Proof: Here
$$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM} = \frac{BC \times AL}{EF \times DM} \dots 1$$

In $\triangle ALB$ and $\triangle DME$

$$\angle ALB = \angle DME \quad \text{Each } 90^\circ$$

and $\angle B = \angle E$ Since $\triangle ABC \sim \triangle DEF$

So, $\triangle ALB \sim \triangle DME$ AA similarity criterion

$$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Since $\triangle ABC \sim \triangle DEF$

Therefore, $\frac{AL}{DM} = \frac{BC}{EF} \dots (2)$

From (1) and (2), we have

$$\frac{\text{Area } (\triangle ABC)}{\text{Area } (\triangle DEF)} = \frac{BC}{EF} \times \frac{AL}{DM} = \frac{BC}{EF} \times \frac{BC}{EF} = \left(\frac{BC}{EF}\right)^2$$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Since $\triangle ABC \sim \triangle DEF$

This implies that,

$$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

37) Sum of the areas of two squares is 544m^2 . If the difference of their perimeters is 32 m, find the sides of the two squares.



OR

A motorboat whose speed is 18km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Answer:

Let the sides of first and second square be x and y . Then,

$$\text{Area of first square} = x^2$$

And,

$$\text{Area of second square} = y^2$$

According to the question,

$$x^2 + y^2 = 544 \quad \dots(1)$$

Now,

$$\text{Perimeter of first square} = 4x$$

And,

$$\text{Perimeter of second square} = 4y$$



According to the question,

$$4x - 4y = 32 \quad \dots(2)$$

From equation (2), we get

$$4(x - y) = 32$$

$$\text{or, } x - y = \frac{32}{4}$$

$$\text{or, } x - y = 8$$

$$\text{or, } x = 8 + y \quad \dots(3)$$

Substituting this value of x in equation(1), we get

$$x^2 + y^2 = 544$$

$$\text{or, } (8 + y)^2 + y^2 = 544$$

$$\text{or, } 64 + y^2 + 16y + y^2 = 544$$

$$\text{or, } 2y^2 + 16y + 64 = 544$$

$$\text{or, } 2y^2 + 16y + 64 - 544 = 0$$

$$\text{or, } 2y^2 + 16y - 480 = 0$$

$$\text{or, } 2(y^2 + 8y - 240) = 0$$

$$\text{or, } y^2 + 8y - 240 = 0$$

$$\text{or, } y^2 + 20y - 12y - 240 = 0$$

$$\text{or, } y(y + 20) - 12(y + 20) = 0$$

$$\text{or, } (y + 20)(y - 12) = 0$$

$$\Rightarrow y + 20 = 0 \text{ or } y - 12 = 0$$

$$\Rightarrow y = -20 \text{ or } y = 12$$

Since side of a square cannot be negative, therefore
 $y = 12$.

Substituting $y = 12$ in equation (3), we get

$$x = 8 + y = 8 + 12 = 20$$

Therefore,

$$\text{Side of first square} = x = 20 \text{ cm}$$

And,

$$\text{Side of second square} = y = 12 \text{ cm}$$

OR

Let the speed of the stream be x km/h.

Therefore, speed of the boat upstream = $(18 - x)$ km/h
and the speed of the boat downstream = $(18 + x)$ km/h.

$$\begin{aligned} \text{The time taken to go upstream} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{24}{18 - x} \text{ hours} \end{aligned}$$



Similarly, the time taken to go downstream = $\frac{24}{18+x}$ hours

According to the question,

$$\begin{aligned} & \frac{24}{18-x} - \frac{24}{18+x} = 1 \\ \text{or, } & \frac{24(18+x) - 24(18-x)}{(18+x)(18-x)} = 1 \\ \text{or, } & 24(18+x) - 24(18-x) = (18+x)(18-x) \\ \text{or, } & 432 + 24x - 432 + 24x = 324 - x^2 \\ \text{or, } & x^2 + 48x - 324 = 0 \end{aligned}$$

Using the quadratic formula, we get

$$\begin{aligned} x &= \frac{-48 \pm \sqrt{48^2 - 4(1)(-324)}}{2} \\ &= \frac{-48 \pm \sqrt{2304 + 1296}}{2} \\ &= \frac{-48 \pm \sqrt{3600}}{2} \\ &= \frac{-48 \pm 60}{2} \end{aligned}$$

$$\text{Therefore, } x = \frac{-48 + 60}{2} \quad \text{or} \quad x = \frac{-48 - 60}{2}$$

$$\Rightarrow x = \frac{12}{2} \quad \text{or} \quad x = \frac{-108}{2}$$

$$\Rightarrow x = 6 \quad \text{or} \quad x = -54$$

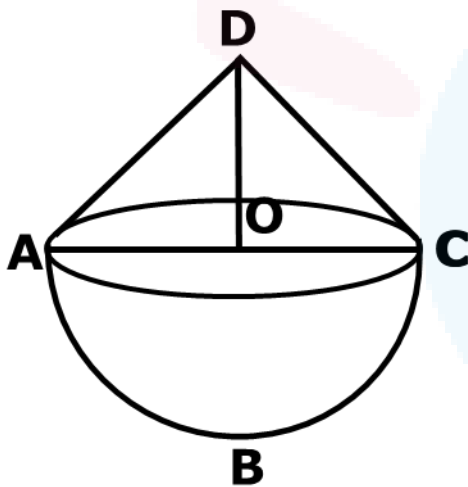


Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -54$. Therefore, $x = 6$ gives the speed of the stream as 6 km/h.

38. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy.

(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

Answer:



Let ABC be the hemisphere and ADC be the cone standing on the base of the hemisphere.

Height of the cone (h_1) = 10 cm (Given)

Radius of the cone (r_1) = 7 cm (Given)

Since the hemisphere is surmounted by the right circular cone of same radius, therefore



Radius of the hemisphere (r_2) = 7 cm

So,

$$\begin{aligned} & \text{Volume of the toy} \\ &= \text{Volume of the cone} + \text{Volume of the hemisphere} \\ &= \frac{1}{3} \pi r_1^2 h_1 + \frac{2}{3} \pi r_2^3 \\ &= \left[\left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 10 \right) + \left(\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \right) \right] \text{cm}^3 \\ &= \left[\frac{1540}{3} + \frac{2156}{3} \right] \text{cm}^3 \\ &= \frac{3696}{3} \text{cm}^3 \\ &= 1232 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} & \text{Area of the coloured sheet required to cover the toy} \\ &= \text{CSA of hemisphere} + \text{CSA of cone} \\ &= 2\pi r_2^2 + \pi r \ell \end{aligned}$$

Where ℓ is the slant height of the cone

$$\begin{aligned} \ell &= \sqrt{r_1^2 + h_1^2} \\ &= \sqrt{7^2 + 10^2} \\ &= \sqrt{49 + 100} \\ &= \sqrt{149} \\ &= 12.2 \text{ cm} \end{aligned}$$

So,

Area of the coloured sheet required to cover the toy

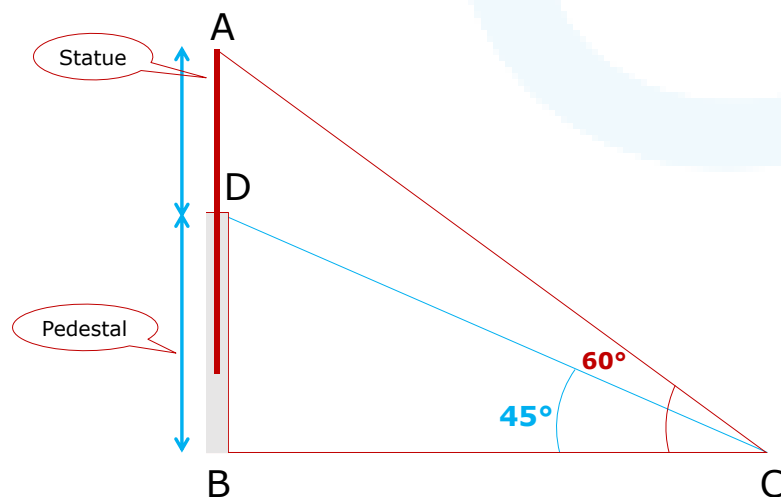
$$= \left(2 \times \frac{22}{7} \times 7 \times 7 \right) + \left(\frac{22}{7} \times 7 \times 12.2 \right) \text{ cm}^2$$
$$(308 + 268.4) \text{ cm}^2$$
$$= 576.4 \text{ cm}^2$$

39. A statue 1.6 m tall, stands on the top of a pedestal.

From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

(Use $\sqrt{3} = 1.73$)

Answer:



Let BD be a pedestal of height x m and AD be a statue of height 1.6 m. The angle of elevation of the top of



pedestal from a point C is 45° and that of point statue from C is 60° .

In the triangle ABC:

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{1.6 + x}{BC} = \sqrt{3}$$

$$\text{Or, } BC = \frac{1.6 + x}{\sqrt{3}} \quad \dots 1$$

In the triangle DBC:

$$\frac{DB}{BC} = \tan 45^\circ$$

$$\text{Or, } \frac{x}{BC} = 1$$

$$\text{Or, } x = BC \quad \dots 2$$

By equations 1 and 2, we get

$$x = \frac{1.6 + x}{\sqrt{3}}$$

$$\text{Or, } \sqrt{3}x = 1.6 + x$$

$$\sqrt{3} - 1 \ x = 1.6$$



$$\begin{aligned}
 \text{Or, } x &= \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\
 &= \frac{1.6 \times 1.73 + 1}{3-1} \\
 &= \frac{1.6 \times 2.73}{2} \\
 &= 2.184\text{m}
 \end{aligned}$$

Therefore, the height of the pedestal is 2.184m.

40) For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age(in years):	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

OR

The distribution given below show the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets	20-60	60-100	100-140	140-180	180-220	220-260
Number of bowlers:	7	5	16	12	2	3

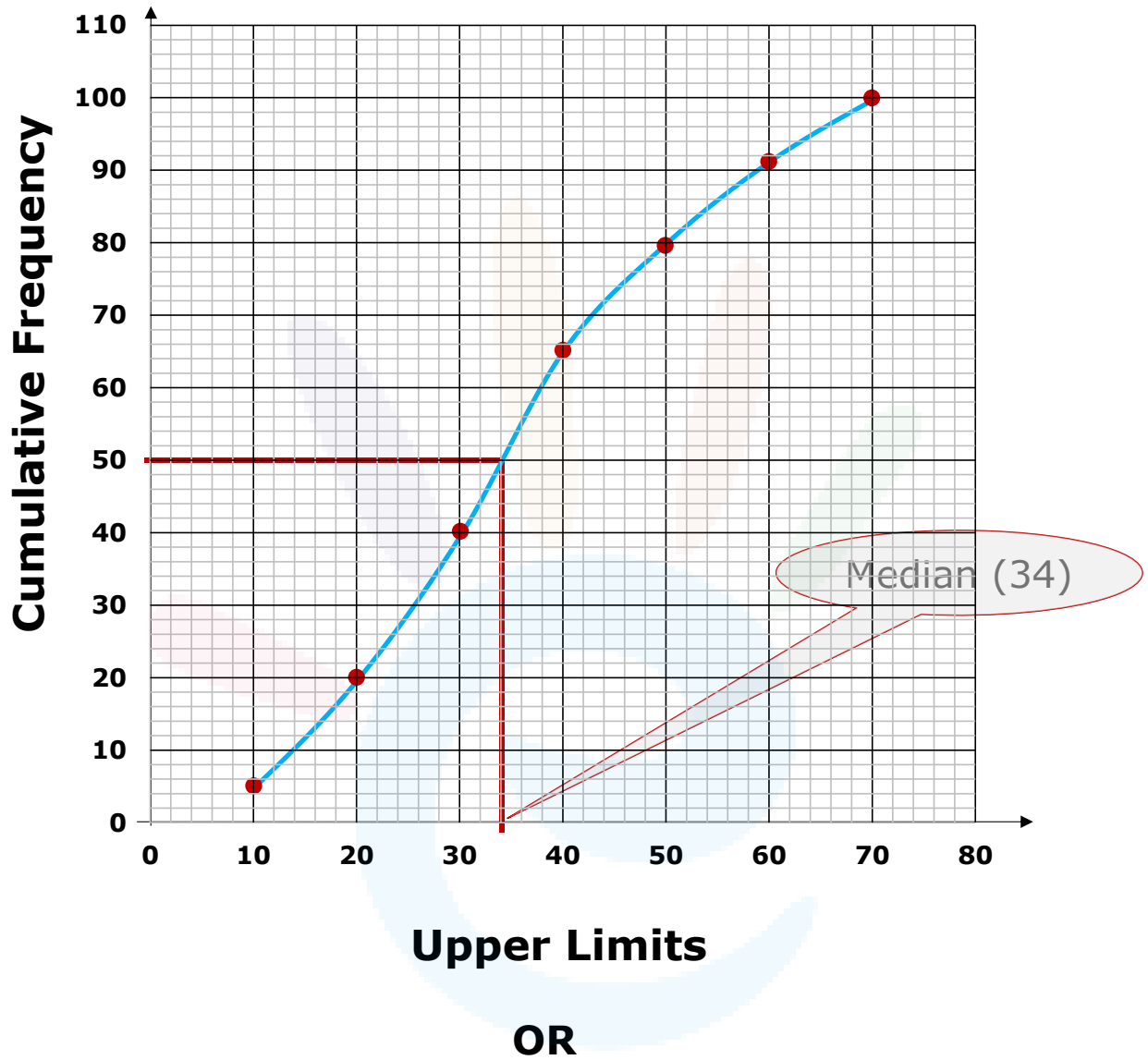
Answer:



Age	Number of Persons (Cumulative frequency)
Less than 10	5
Less than 20	$5 + 15 = 20$
Less than 30	$20 + 20 = 40$
Less than 40	$40 + 25 = 65$
Less than 50	$65 + 15 = 80$
Less than 60	$80 + 11 = 91$
Less than 70	$91 + 9 = 100$

Age	No. of Persons (f)	Cumulative frequency (cf)
0 - 10	5	5
10 - 20	15	20
20 - 30	20	40
30 - 40	25	65
40 - 50	15	80
50 - 60	11	91
60 - 70	9	100

Plot the points (10, 5), (20, 20), ..., (70, 100) on a graph paper.





Class interval	No. of bowlers f_i	Class mark x_i	$f_i x_i$
20 – 60	7	40	280
60 – 100	5	80	400
100 – 140	16	120	1920
140 – 180	12	160	1920
180 – 220	2	200	400
220 – 260	3	240	720
Total	$\sum f_i = 45$		$\sum f_i x_i = 5640$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{5640}{45} = 125.33$$

Number of wickets	Number of bowlers	Cumulative Frequency
20 – 60	7	7
60 – 100	5	12
100 – 140	16	28
140 – 180	12	40
180 – 220	2	42
220 – 260	3	45



$$n = 45$$

$$\Rightarrow \frac{n}{2} = \frac{45}{2} = 22.5$$

$$\text{Median class} = 100 - 140$$

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$l = 100, \frac{n}{2} = 22.5, cf = 12, f = 16, h = 40$$

$$\begin{aligned} \text{Median} &= 100 + \frac{22.5 - 12}{16} \times 40 \\ &= 100 + 26.25 \\ &= 126.25 \end{aligned}$$